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A Microgenetic Study of Simple Division

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How simple division strategies develop over a short period of time was examined with a microgenetic study. Grade 5 students (mean age = 10 years, 3 months) solved simple division problems in 8 weekly sessions. Performance improved with faster and more accurate responses across the study. Consistent with R. S. Siegler's (1996) overlapping waves model, strategies varied in their use. Direct retrieval increased, retrieval of multiplication facts remained steady, and addition facts, derived facts, and special tricks marginally decreased. Consistent with previous research, multiplication fact retrieval was the most common strategy, although it was slower and more error prone than direct retrieval. Strategy variability within and across individuals was striking across all of the sessions and underscores Siegler's (1996) assertion that development is in a constant transitional state.

Keywords: arithmetic, division, strategy, microgenetic

The development of arithmetic strategies has the potential to yield critical information on the development of both arithmetic skills and knowledge and of cognitive development in general. Most research on arithmetic strategies has focused particularly on simple addition and multiplication. Although there is little research-based evidence, it is believed that similar problem-solving strategies are used on simple subtraction and division problems (Geary, 1994). Researchers have started to examine the strategies that adults and children use to solve simple division problems and have found that the pattern of division strategy choice does not correspond to the patterns of strategies used on the other arithmetic operations (LeFevre & Morris, 1999; Mauro, LeFevre, & Morris, 2003; Robinson, Arbuthnott, & Gibbons, 2002; Robinson et al., 2006). Specifically, on division problems, both adults and children rely much less on direct retrieval and much more on backup strategies than they do for the other arithmetic operations. However, how children's division strategies develop and change over relatively short periods of time is unknown.

In all arithmetic operations, the goal is to eventually be able to automatically retrieve answers to simple arithmetic problems directly from memory as retrieval is faster and more efficient than other strategies (Siegler, 1996; Siegler & Shipley, 1995; Siegler & Shrager, 1984). Siegler's models posit that if retrieval is not a viable option or has failed, then backup strategies (e.g., counting, using derived facts) will be used to attain an answer. As individuals become more skilled, retrieval will be used more frequently, and the backup strategies will become mostly obsolete (Ashcraft, 1982; Roussel, Foyal, & Barouillet, 2002). Most research has shown that certainly by adulthood retrieval is used almost exclusively (e.g., Campbell & Timm, 2000; although see LeFevre et al., 1996). In childhood, retrieval is also commonly used (Kerkman & Siegler, 1997), at least for the operations of addition, subtraction, and multiplication.

Although much research supports the idea that retrieval is the final goal of development, Siegler's (1996) overlapping waves model proposes that during development children rely on a variety of strategies and that this variability can be prolonged. Children do not simply use one strategy until a more effective one comes along. Children use old and new strategies concurrently, with strategies waxing and waning over time. The overlapping waves model is appropriate to current research on the development of simple division strategies, which has shown that children use a variety of strategies across a long period of development (Robinson et al., 2006). However, this research has also shown that in contrast to the other arithmetic operations, on simple division problems children do not make the switch to using retrieval as their primary problem-solving process and instead rely increasingly on a different strategy, using related multiplication facts, a strategy that is frequently used by adults as well (Robinson et al., 2002).

LeFevre and colleagues (LeFevre & Morris, 1999; Mauro et al., 2003) have referred to the use of retrieving related multiplication facts when solving simple division problems as "mediated retrieval," and Robinson and colleagues (Robinson et al., 2002, 2006) have referred to this as "multiplication retrieval" to be more specific about what information is being retrieved. Children solving simple division problems use mediated retrieval involving addition, subtraction, and multiplication facts, although they predominantly use multiplication facts (Robinson et al., 2006).

The development of division strategies follows a slow, progressive pattern. In contrast to the other operations, even by Grade 7 children are not relying predominantly on direct retrieval. Instead, Robinson et al. (2006) found that the trend is for Grade 4 students

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to rely heavily on adding the divisor until the dividend is reached (more than 50% use). In Grade 5 and increasingly in Grade 6, students start to switch to multiplication retrieval and by Grade 7 this strategy predominates (more than 70% use). Surprisingly, direct retrieval remains constant (and infrequent) across all grades (less than 20% use in all grades). Thus, the period of greatest change appears to be in Grade 5 when multiplication retrieval increases by more than 30%. To investigate the development of simple division strategies in more detail, we conducted a microgenetic study with Grade 5 students. The microgenetic method allows for the examination of detailed information about the development of a strategy or skill as individual development over a short time period (Siegler, 2006; Steiner, 2006). The microgenetic method has been identified as useful for obtaining detailed information about change and development (Siegler, 2002, 2006; Siegler & Crowley, 1991).

Method

Participants

Thirty-two Grade 5 students (16 boys and 16 girls; mean age = 10 years, 3 months) were included in this study. Participants were drawn from a large Canadian city and were predominantly White and middle class. The study involved eight weekly sessions, which took place in the first half of the school year.

Materials and Procedure

Once a week for 8 consecutive weeks, participants solved 16 simple division problems. As is not unusual in developmental research in which data collection occurs in schools, participants were tested by one of the three experimenters in one of three different testing locations within each school over the course of the study. The problems were selected from all division problems (except ties) between $4 \div 2$ and $81 \div 9$ (i.e., from the 2×2 to 9×9 multiplication table). Half of the problems in each session were small (dividend < 25, e.g., $16 \div 2$) and half were large (dividend > 25, e.g., $72 \div 9$). Problems were chosen so that all were used equally as often across sessions and the same problems were not presented in consecutive sessions. Problems were presented one at a time on 8.5-in. × 11-in. (21.59-cm × 27.94-cm) white paper. There were eight problems per page, and only one problem at a time was revealed. Errors, solution latencies, and verbal reports of solution strategy were collected for each problem. Solution latencies were collected using a stopwatch. As is typical with most of the research on children's arithmetic strategies, after each problem participants were asked how they had solved each problem. Children's verbal reports have been shown to be a veridical and useful source of information (e.g., Robinson, 2001; Siegler, 1987, 1989). Each session was conducted individually and videotaped.

Results

All of the significant differences had an alpha level of .05. Boys and girls were collapsed together as no significant results involving gender were found.

Changes in Errors and Solution Latencies

Errors and solution latencies were examined to determine whether there were overall changes in performance from the beginning to the end of the microgenetic study. Median solution latencies on correct trials were calculated for each problem size. One participant who did not have any correct solution latencies in one of the cells was eliminated from the solution latency analysis. Two 2 (problem size: small and large) × 2 (session: 1 and 8) analyses of variance (ANOVAs) were run on the error and solution latency data, respectively. Participants' errors and their solution latencies decreased from the first to the last session, $F(1, 31) = 19.86$, $MSE = 284.15$, and $F(1, 30) = 19.05$, $MSE = 16.52$ (errors are shown in the top panel and solution latencies in the bottom panel of Figure 1). Errors dropped from 18.9% to 5.7%, and solution latencies decreased from 7.1 s to 3.9 s. Performance was also better on small problems than on large problems, $F(1, 31) = 9.93$, $MSE = 307.30$, for errors and $F(1, 30) = 17.83$, $MSE = 19.26$, for latency. Errors occurred on 7.4% of small problems and 17.2% of large problems, and solution latencies were 3.8 s on small problems and 7.1 s on large problems. Overall, the pattern of change from the beginning to the end of the microgenetic study was one of gradual improvement in performance, particularly on large problems. This pattern is consistent with a gradual increase in more efficient problem-solving solution strategies.

Changes in Strategy Choice

Strategies were placed into five main categories developed in previous studies of simple division strategies (Robinson et al., 2002, 2006). After data collection was completed, two raters coded 10% of the sessions; interrater reliability was 93.8%. First, retrieval was defined as getting the answer directly from memory. Second, multiplication was defined as retrieval of the related multiplication fact (e.g., $27 \div 9$: ? × 9 = 27). Third, addition was defined as adding the divisor until the dividend was reached (e.g., $27 \div 9$: 9 + 9 + 9). Fourth, derived fact or special trick was defined as either breaking the problem down into smaller components (e.g., $35 \div 5$: $30 \div 5 = 6 + 1$) or using a problem-specific strategy (e.g., $56 \div 7 = 8$ because 5, 6, 7, 8). Fifth, other strategies included addition retrieval (defined as retrieving the related addition fact), counting subtraction (defined as subtracting the divisor from the dividend), and guessing, but these strategies were used so infrequently (less than 5% total) that they were excluded from the reported analyses.

As seen in Figure 2, overall strategy use for direct retrieval (top left panel), multiplication retrieval (top right panel), addition (bottom left panel), and derived fact or special tricks (bottom right panel) across the eight sessions clearly indicates a waxing and waning of strategy use as predicted by Siegler's (1996) overlapping wave model. Four 2 (session: 1 and 8) × 2 (problem size) ANOVAs were performed for each of the four strategy categories to get some sense of how the strategies changed over time, although the strategy frequencies are not independent. Direct retrieval increased, $F(1, 31) = 12.39$, $MSE = 297.97$ (14.8% to 25.6%). Multiplication remained steady, $F(1, 31) = 2.55$, $MSE = 1{,}058.43$, although the means increased nonsignificantly (29.5% to 38.7%). Counting addition and derived fact or special trick use had marginally significant decreases across sessions, $F(1, 31) = 3.17$,

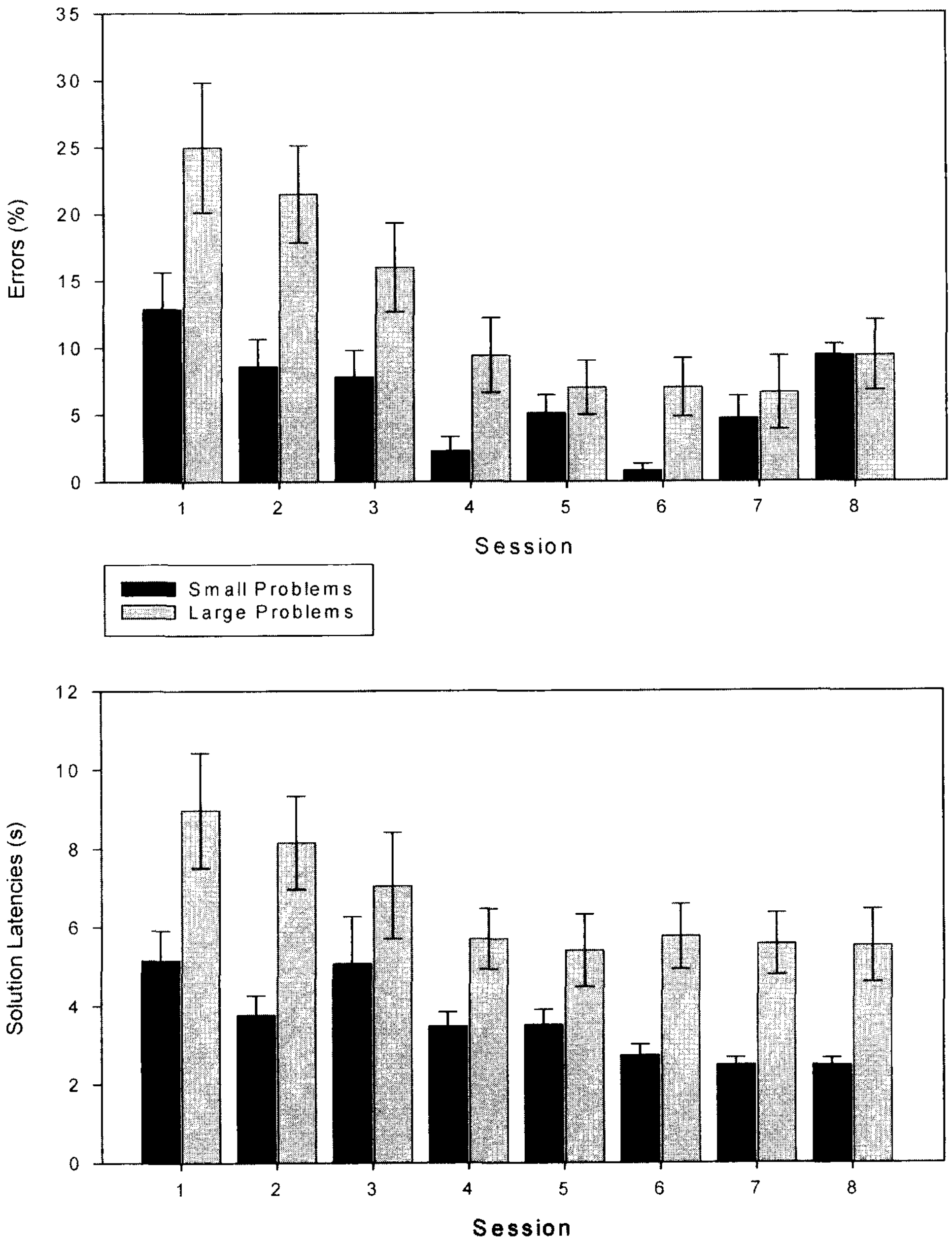

Figure 1. Mean errors (in percentages) and solution latencies (in seconds) with standard error bars on small and large problems across sessions.

MSE = 419.57, p = .085, and $F(1, 31)$ = 3.10, *MSE* = 226.81, p = .088, respectively, with counting addition decreasing from 27.5% to 21.1% and derived fact or special trick decreasing from 11.1% to 6.4%. Only one significant problem-size effect was found with the least frequent category, derived facts or special tricks, which was reported more on large problems than on small problems, $F(1, 31)$ = 19.67, *MSE* = 109.47 (12.9% vs. 4.7%).

Strategy Report Validity

Previous studies of performance on simple division problems have found that participants are able to veridically report their solution processes (Robinson et al., 2002, 2006). If participants were using retrieval as reported, then their solution latencies should be comparatively fast and error free. On the basis of research by Robinson and colleagues, it is known that multiplication is also a fast and accurate solution process. Derived fact or special trick strategies are intermediate strategies that, although slower than the first two strategies, are still stronger than counting addition. Errors and solution latency patterns were exactly as predicted. Collapsing across all trials for all participants, retrieval was fast and accurate (2.3 s and 2.4% errors), multiplication performance was almost as strong (3.4 s and 6.5%), and derived

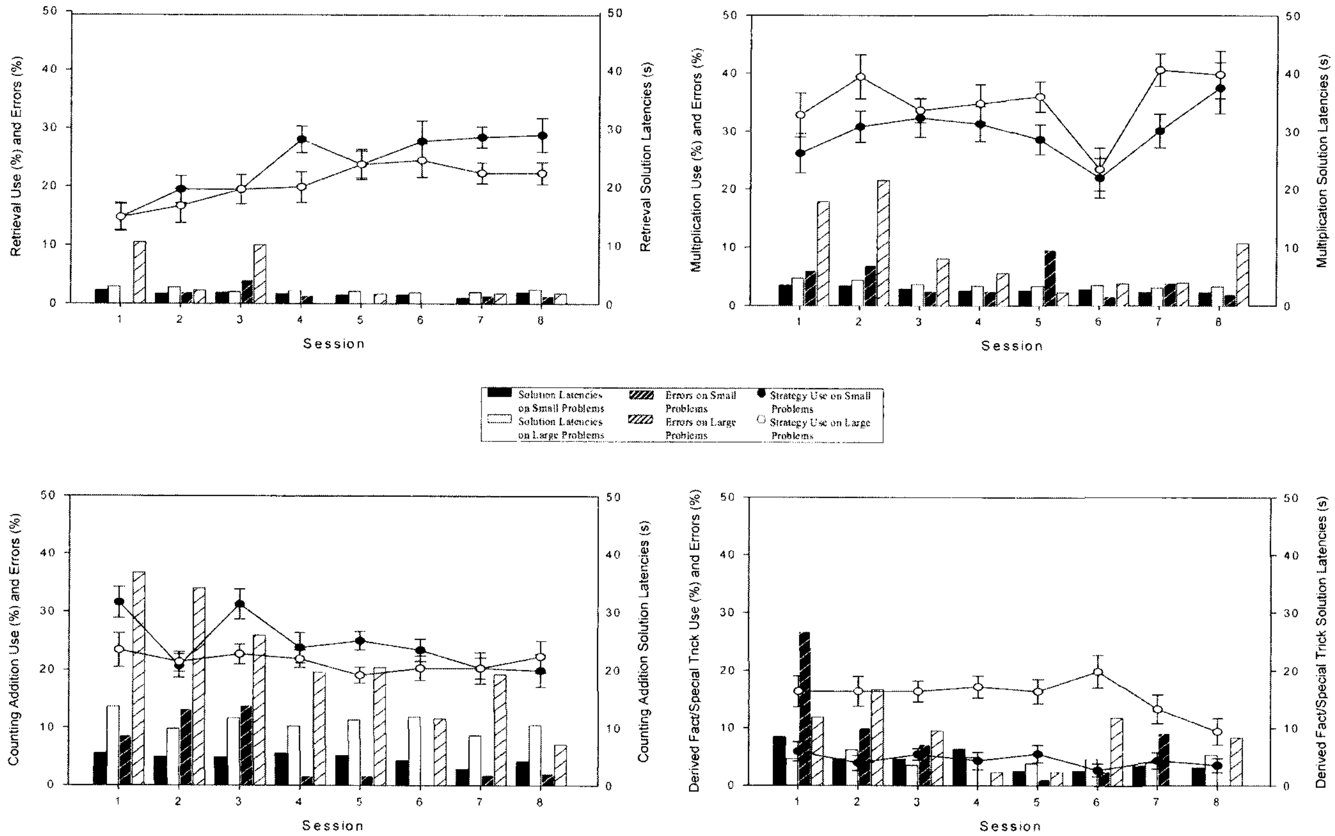

Figure 2. Mean strategies (in percentages with standard error bars) and associated errors (in percentages) and solution latencies (in seconds) for retrieval, multiplication, addition, and derived fact or special trick strategies on small and large problems for each session.

facts or special tricks (7.0 s and 7.7%) and counting addition (10.1 s and 13.3%) were progressively slower and more error prone. Thus, there is converging evidence for the veridicality of the verbal reports of solution strategies provided by the participants.

Strategy Variability

Although retrieval increased across sessions and although multiplication remained relatively steady and counting addition and the derived fact or special trick marginally decreased, closer examination of all sessions revealed that strategy variability was striking across the entire study. On average, by the end of the study participants had used an average of 3.0 strategies (out of a possible 5). Within each session, 68.8% of the participants used three or more strategies. A closer examination of individual participants' strategy variability showed no clear groupings of strategy development. To demonstrate this variability across individuals, five representative participants' multiplication use was charted (see Figure 3). The possibility exists that the variability in testing locations and experimenters could perhaps have contributed to the individual variability in strategy use. The data from these participants were chosen as they had low, medium, or high multiplication use and low, medium, or high variability across sessions. This variability underscores Siegler's (1996) assertion that children's strategy use in simple arithmetic is highly variable both within individuals and across groups.

Multiplication Fact Retrieval Versus Direct Retrieval

By the end of middle childhood and definitely by adulthood, retrieval should be the most commonly used problem-solving approach (Ashcraft, 1982; Campbell & Timm, 2000). However, for simple division, retrieval does not appear to become the predominant problem-solving strategy for children (Robinson et al., 2006), and even adults do not use it as frequently as with other operations (Campbell & Xue, 2001). This may be due to a backup strategy, multiplication, remaining, unlike most other backup strategies, a strong problem-solving competitor. If a backup strategy is almost as fast and accurate a process as direct retrieval, then this strategy will not be so quickly or easily discarded. Two two-way ANOVAs (Strategy × Problem Size) were performed on the percentage of accurate responses and on the median solution latencies for multiplication and retrieval trials for each of the problems (collapsed across sessions). Retrieval was both more accurate than multiplication (although there may be ceiling effects), $F(1, 63) = 21.69$, $MSE = 50.41$ (2.4% vs. 6.5% errors), and faster, $F(1, 63) = 53.24$, $MSE = 1.34$ (2.3 s vs. 3.4 s). As reported previously, retrieval was both faster and less error prone than

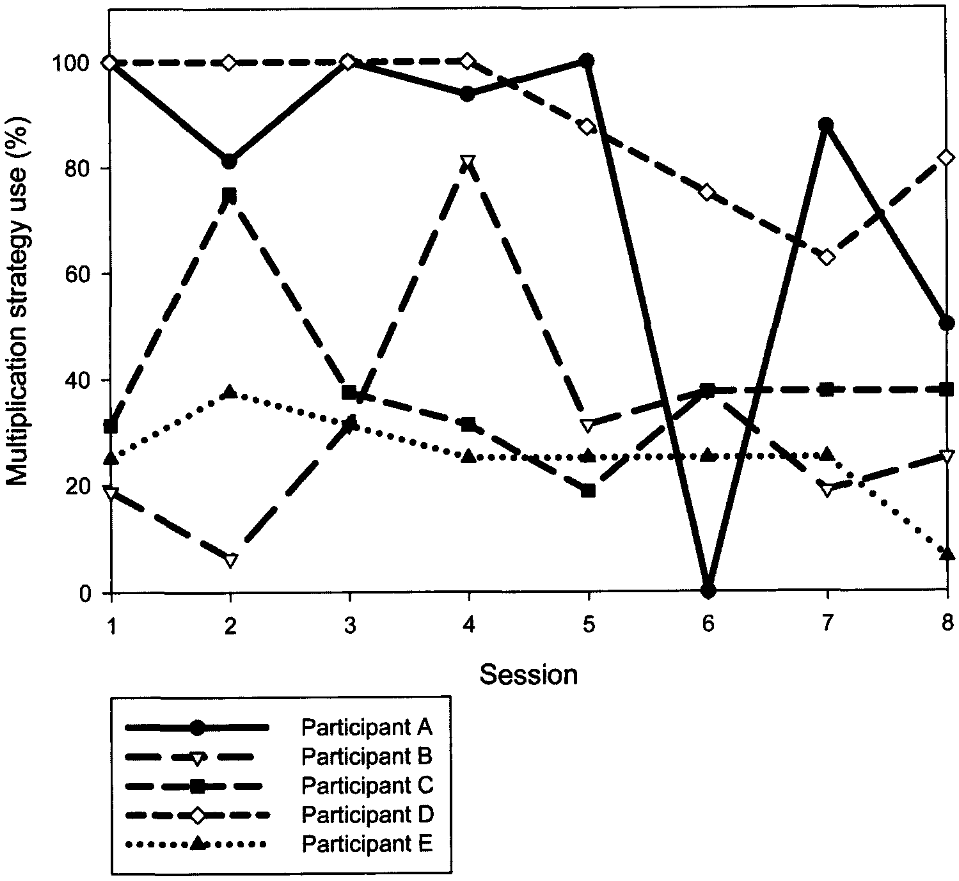

Figure 3. Examples of representative patterns of individual variability on the percentage of multiplication strategy use across sessions.

multiplication, but in the ANOVA on solution latencies, there was also an interaction between problem size and strategy, $F(1, 63) = 13.25$, $MSE = 0.96$. Retrieval was faster than multiplication on both problem sizes; however, although there was a problem size effect on multiplication trials, retrieval was as fast on small problems as on large problems (Tukey Honestly Significant Difference = .52). However, note that the trend was always for retrieval solution latencies to be longer on large problems than on small problems in all sessions (see Figure 2). No Strategy × Size interaction was found for errors, presumably because of ceiling effects.

So, although retrieval is clearly superior to the backup strategy of multiplication, it is also clear that multiplication, although slower than direct retrieval, is the most efficient of the backup strategies, as overall strategy solution latencies were 10.1 s for counting addition and 7.0 s for derived facts or special tricks.

In most instances of multiplication retrieval, participants directly retrieved the related multiplication fact, but sometimes participants reported retrieving several related multiplication facts (e.g., for 72) 9, retrieve 9 × 6, then 9 × 7, then 9 × 8). This occurred on 24.2% of multiplication retrieval trials, and 84.4% of participants reported using repeated multiplication at least once. In such cases, solution latencies would be longer and errors more likely, so analyses were redone with these trials removed. Retrieval still remained significantly more accurate, $F(1, 63) = 7.20$, $MSE = 56.99$ (97.6% vs. 95.1%), and faster than multiplication, $F(1, 63) = 18.29$, $MSE = 1.32$ (2.3 s vs. 2.9 s), but the means were closer. No interaction involving problem size was found on solution latencies, indicating that multiplication was as fast on small problems as on large problems when "direct" multiplication was used, just as was found on direct retrieval trials. Thus, when participants directly retrieved either multiplication or division facts, they were fast, irrespective of problem size.

General Discussion

Division strategies showed varying frequencies during the microgenetic study, consistent with Siegler's (1996) overlapping wave model. Retrieval increased, multiplication retrieval remained steady, and the addition and derived fact or special trick strategies marginally decreased. Of the strategies, multiplication was the most commonly reported across the whole study.

This is the first study to examine both general problem-size effects and performance problem-size effects associated with each strategy in children's division. The expected overall problem-size effect of fewer errors and faster solution latencies on small problems than on large problems was found. Problem-size effects were also expected and found in strategy use (e.g., more retrieval use on small problems, more use of a backup strategy such as multiplication on large problems). However, there was no problem-size effect for percentage of strategy use for either retrieval or multiplication use, but the trends were in the expected directions. The examination of the associated performance data of these strategies indicated that retrieval was as fast and accurate on small problems as on large problems, but multiplication was faster and less error prone on small problems than on large problems. The data suggest that when children know a division fact well, problem size does not matter.

Most research indicates that direct retrieval is the problem-solving approach of choice. In this study, direct retrieval was the most efficient strategy, and yet multiplication retrieval remained competitive, particularly on trials for which participants were able to immediately retrieve the related multiplication fact. This frequent and continued reliance on the retrieval of multiplication facts, also found by Robinson et al. (2006) with children from Grades 4 through 7 and to a lesser extent with adults (LeFevre & Morris, 1999; Mauro et al., 2003; Robinson, 2001), has implications for how simple arithmetic facts are learned. The performance difference between the backup strategies suggests that the backup strategy of multiplication is efficient enough that there is less incentive to abandon it (cf. the counting addition strategy). Although the statistical analyses demonstrated that performance on multiplication trials was slower and more error prone than retrieval, in practice this difference in performance (less than 1 s on direct multiplication trials compared with direct retrieval trials) may have been too negligible for participants to perceive retrieval as a clearly superior strategy choice. Instead, participants' confidence in problem-solving success would be higher for the more familiar strategy of multiplication retrieval with acceptable performance-related costs. Siegler's models of strategy choice (Siegler & Shipley, 1995; Siegler & Shrager, 1984) support this notion that on different problems and different strategies, children have different confidence criteria, and it then follows that children also have different confidence criteria for different strategies on different operations. Given that multiplication and division are very similar in performance, children may be most comfortable, except on problems for which they have high associative strengths between the problem and the correct response (which tend to lead toward higher direct retrieval use; Siegler & Shipley, 1995), in adopting a well-practiced, comfortable strategy of retrieving related multiplication facts. That is, children are being adaptive in their strategy choice. There is also evidence in the adult literature that division performance continues to be mediated by multiplication into the adult years (Campbell, 1997, 1999; LeFevre & Morris, 1999; Mauro et al., 2003; Rickard, 2005; Rickard & Bourne, 1996; Rickard, Healy, & Bourne, 1994). Overall, the shift to retrieval as the problem-solving strategy of choice is a long and protracted one, with retrieval only beginning to gain predominance in adulthood.

Reasons for the continued reliance on the associated operation of multiplication to solve division problems need to be further investigated. Although it might be expected that on subtraction problems there might be a similar pattern of retrieval of addition facts, this has not been found (Robinson, 2001). Some factors might explain this dependence on related multiplication facts. First, multiplication facts are typically well practiced and therefore easily accessible (i.e., learning the times tables). Second, division instruction often focuses less on learning simple division facts and more on the learning of long division skills. Long division relies heavily on associated multiplication facts. In contrast, on complex subtraction problems subtraction skills, not addition skills, are necessary. Third, Rickard (2005) has hypothesized that multiplication memory involves a reverse association from products to factors, and this reverse association fosters the use of the multiplication strategy on division problems. In contrast, Campbell, Fuchs-Lacelle, and Phenix (2006) have found no evidence for such a reverse association between addition and subtraction.

Pronounced and prolonged strategy variability was found across all sessions. Although the participants solved similar problems for 8 consecutive weeks, variability did not decrease, supporting the view that transition periods in strategy development are often prolonged (Siegler, 1996) and that even on simple arithmetic problems, variability within and between individuals is marked. The ongoing variability found in this study with children who had been solving division problems for at least 2 years also supports the conclusion by Robinson et al. (2006) that division has unique characteristics that distinguish it from the other arithmetic operations. It is possible that the variability may be because all operations can be used to solve a division problem, thereby enhancing the use of a more diverse set of backup strategies.

Conclusion

Of the four arithmetic operations, division is the most difficult, and division skills follow a different developmental trajectory. The results of this study build on the results from earlier studies of children's division skills and further current understanding of how these skills develop from childhood to adulthood. The results of this study yielded three important findings. First, this study demonstrates that individuals' simple division strategies develop very slowly and gradually and that this pattern of development is divergent from the developmental patterns for the other arithmetic operations. Thus, this study provides further evidence for the distinctiveness of strategy use on simple division problems as well as support for Siegler's (1996) overlapping waves model. The microgenetic approach shows that over a relatively short period of development, performance on simple division problems remains distinct from performance on addition, subtraction, and multiplication problems. Participants used a variety of strategies and relied heavily on the other operations to help them problem solve. Unlike the other operations, participants did not move toward relying predominantly on direct retrieval. Retrieval did increase over the span of the study, suggesting that more practice and exposure to division problems over a short period of time helped consolidate division facts.

Second, the variability in division strategies was a striking finding in this study. Although the results of this study were consistent with Siegler's (1996) assertion that variability in problem-solving strategies can be prolonged and marked, this study provided evidence of just how variable strategy choices can be both within and across individuals. Strategy choices ebbed and flowed throughout the microgenetic period. Siegler noted that children have multiple ways of thinking and that "all of development is a transitional period" (p. 112). Our closer examination of the development of simple division strategies across a relatively short period of time provides evidence of the adaptiveness and variability in children's thinking.

Third, as far as children are concerned, multiplication is a division problem-solving strategy that is as good as retrieval. Siegler and Shipley (1995) proposed that the strongest strategy, retrieval, will eventually win when problem solving. However, despite the statistically significant differences found in performance between the two strategies, for children the difference is not enough to cause them to discard multiplication as a backup strategy as quickly as they do the backup strategies for solving addition, subtraction, and multiplication problems. Therefore, on simple division problems, the contest appears to be unusually long and protracted, with retrieval only beginning to gain clear predominance in adulthood. Overall, the distinctiveness of division problem-solving strategies, the substantive inter- and intraindividual variability in strategy choice, and the strength and steadfastness of backup strategy use are all central to understanding further development of children's arithmetic skills.

Résumé

Cette étude microgénétique examine comment les stratégies de division simple se développent sur une courte période. Des élèves de 5e année (âge moyen = 10 ans et 3 mois) ont résolu des problèmes de division simple durant huit sessions hebdomadaires. La performance s'est améliorée au fil de l'étude, les réponses étant plus rapides et plus précises. Conformément au modèle de chevauchement ondulatoire de Siegler (1996), une variation quant à l'utilisation des stratégies a été observée. La récupération directe a augmenté, la récupération des tables de multiplication est demeurée constante, et le recours aux tables d'addition et dérivées, ainsi que les trucs spéciaux ont marginalement diminué. Conformément aux recherches antérieures, la récupération des tables de multiplication a été la stratégie la plus commune, bien qu'elle ait été plus lente et associée à plus d'erreur que la récupération directe. La variabilité des stratégies intra et inter individuelles était marquante durant toutes les sessions et appuie la proposition de Siegler (1996) selon laquelle le développement est constamment dans un état transitoire.

Mots-clés : arithmétique, division, stratégie, microgénétique

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